Estimation of process variables

DESCRIPTION

Field of the Invention

The invention relates to the field of process control. It relates to a method, computer program and data processing system for the estimation of process variables as described in the preamble of claims 1, 8 and 9, respectively.

Background of the Invention

Mathematical modelling is an essential tool in modern industry. It makes possible a whole range of procedures for process control, planning, scheduling, optimisation, monitoring etc., where processes considered are technical or physical processes such as industrial production processes, power

plants, mechatronic devices etc. The success of the above procedures hinges on the ability of the mathematical models to represent the physical reality at a needed degree of accuracy. The mathematical models incorporate a number of parameters. Some of these parameters have a physical meaning, e.g. mass, flow, temperature, force. Other parameters are coefficients of given parameterised functional dependencies among physical properties of the process. For example, in a turbomachine, i.e. a compressor or a turbine, such dependencies are expressed by polynomials describing relationships such as

mass flow =f1(vane position, pressure ratio, rotor speed, etc)
efficiency=f2(temperature, mass flows, pressures, etc)

where f1 and f1 are static non-linear functions of the bracketed variables, which represent measured or estimated process states in the machine. Graphical representations of these functions are so called "compressor maps". Coefficients of polynomials replacing these functions are usually computed via least square fitting of experimental data. Usually only manufacturers have enough data to carry out the fitting at all relevant conditions. Moreover, the process of fitting is costly and cumbersome. During operation, not in a test set-up but at an arbitrary installation, it is difficult to measure accurately all the values entering the relationships f1 and f2 and therefore to deduce a suitable approximation.

In order to obtain estimates of the relationships on-line, i.e. during process operation, estimation techniques have been developed: The "Kalman Filter" (KF)

for linear systems emerged in the 1960's and has been developed into a mature technology for reconstruction of the state of a given linear plant. The "Extended Kalman Filter" (EKF) is used when the KF formulae are applied to estimation of the internal states of nonlinear plants. In this case, linearisations of the true system equations, at current estimates, are used. If parameters entering the plant are unknown then the "State Augmented Extended Kalman Filter" (SAEKF) is used. The SAEKF provides estimates of the polynomial coefficients representing unknown relationships within the mathematical process model, such as the functions f1 and f2 mentioned above. For example, f2 could be represented by a third-order polynomial

$$\text{efficiency } e = \mathbf{k}_0 + \mathbf{k}_1 \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \mathbf{k}_2 \begin{bmatrix} x_1^2 \\ x_2^2 \\ \vdots \\ x_n^2 \end{bmatrix} + \mathbf{k}_3 \begin{bmatrix} x_1^3 \\ x_2^3 \\ \vdots \\ x_n^3 \end{bmatrix}$$

where $k_0 \ldots k_3$ are vectors with polynomial coefficients to be estimated, and $x_1 \ldots x_n$ are process states representing vane position, pressure, speed, etc. The complete estimation of a physical property of the process such as the efficiency e is done according to the block diagram of figure 1. The polynomial relation shown above is represented by the right hand side of the block diagram, comprising multiplication and addition blocks. A state vector according to the SAEKF comprises the coefficient vectors $k_0 \ldots k_3$ as extended state variables. In the block diagram, these states are represented by outputs of integrators having a constant input of zero. The SAEKF algorithm modifies the states, providing updated values for the coefficients $k_0 \ldots k_3$, which are

then combined with the states x_1 ... x_n to compute an estimate of the efficiency e.

However, because the coefficients k_0 ... k_3 lack physical meaning, their values are very sensitive to variations in the data and are not adequate for extrapolation to conditions other than the ones they were generated from. Furthermore, the polynomial parameterisation restrains the degrees of freedom, which might be crucial for an adequate description of the physical phenomena under investigation. For instance, the real physical relationships might incorporate discontinuities, which would not be adequately represented if the experimental data is represented by polynomial fitting.

Description of the Invention

It is therefore an object of the invention to create a method, computer program and data processing system for the estimation of process variables of the type mentioned initially, which overcomes the disadvantages mentioned above.

These objects are achieved by a method, computer program and data processing system for the estimation of process variables according to the claims 1, 8 and 9.

In the inventive method for estimating a value of a vector of variables **p** in a mathematical model representing a physical process, the vector **p** represents

one or more properties of the process where these properties are representable by a function of the state vector, and are estimated as an augmented state by the SAEKF.

That is, the process properties such as mass flow or efficiency *themselves* are estimated, and not polynomial coefficients for computing the variables from the state! The surprising result is that this approach works and that the abovementioned disadvantages are overcome.

In a preferred variant of the invention, the physical process comprises a turbomachine, and the vector of variables p represents at least one of an efficiency or a mass flow rate of the turbomachine.

In a further preferred variant of the invention, the physical process comprises a heat exchanger, and the vector of variables p represents at least one heat transfer coefficient of the heat exchanger.

In a further preferred variant of the invention, the physical process comprises a mating gear transmission, and the vector of variables p represents a backlash and spring function.

The computer program for the estimation of process variables according to the invention is loadable into an internal memory of a digital computer, and comprises computer program code means to make, when said computer program code means is loaded in the computer, the computer execute the

method according to the invention. In a preferred embodiment of the invention, a computer program product comprises a computer readable medium, having the computer program code means recorded thereon.

A data processing system for the estimation of process variables comprises means for carrying out the steps of the method according to the invention. In a preferred embodiment of the invention, the data processing system is an apparatus comprising a data processor, a memory coupled to the processor and computer program code means stored in said memory, where said computer program code means, when executed by the processor, causes the method according to the invention to be executed.

Further preferred embodiments are evident from the dependent patent claims.

Brief Description of the Drawings

The subject matter of the invention will be explained in more detail in the following text with reference to preferred exemplary embodiments which are illustrated in the attached drawings, in which:

Figure 1 shows a block diagram for the computation of a process property according to the state of the art;

Figure 2 shows a block diagram for the computation of a process property according to the invention;

Figure 3 shows a block diagram representing the computation of process variables in a flow process;

Figure 4 schematically shows process variables of a heat transfer process;

Figure 5 schematically shows a mechanical system with backlash;

Figure 6 shows a backlash torque τ_k with memory effect;

Figure 7 shows a polynomial approximation to a backlash and spring function; and

Figure 8 shows a continuous model approximation of friction torque.

The reference symbols used in the drawings, and their meanings, are listed in summary form in the list of reference symbols. In principle, identical parts are provided with the same reference symbols in the figures.

Detailed Description of Preferred Embodiments

The invention shall first be explained in general terms as a special embodiment of a State Augmented Extended Kalman Filter (SAEKF), and then as applied to specific examples, i.e. a turbomachine, heat exchanger and gear backlash.

The well-known Kalman Filter estimates a system state of a dynamic system that is representable by state-space representation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u})$$

with state vector $\mathbf{x}$ and input vector $\mathbf{u}$. In the SAEKF, the state is augmented by variables $\mathbf{p}$ to be estimated, and the state-space model underlying the SAEKF is

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p}) \\ 0 \end{bmatrix} + \nu \qquad (1)$$

where $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p})$ represents a known dependency of the change $\dot{\mathbf{x}}$ in system state from the state vector **x**, the measured values **u** and the vector of variables **p**, and ν represents a vector of noise disturbances. The original state vector **x** shall be referred to as state vector, whereas the vector of variables **p** is denoted as augmented state. The combined vector

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix}$$

shall be called *complete state*. Estimates of the states are denoted by $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$. In a preferred embodiment of the invention, the variables **p** to be estimated correspond to physical properties of the process, i.e. to properties whose value have an interpretation in terms of the process, as opposed to e.g. coefficients of the polynomial approximations. In the simplest case, the vector **p** is a scalar.

The computation of the state estimate along with associated covariance matrices is done according to a suitable implementation of the known SAEKF approach, as shown e.g. in Robert Stengel, "*Optimal control and estimation*", Dover Publications, 1994.; and in C. Bohn, "*Recursive Parameter Estimation for Nonlinear Continuous Time Systems through Sensitivity Model Based Adaptive Filters*", PhD Dissertation, University of Bochum, Germany, 2000. The Extended Kalman Filter is closely related to a wide family of system identification methods called Recursive-Prediction-Error-Methods (RPEM). The main difference between the methods in this class lies in the computation of the gradients of the optimisation criterion, see for example L. Ljung, ""System

Identification: Theory for the User", 1999, Chapter 11. Hence, the concept of replacing polynomial approximations with one single augmented state also applies in the more general setting of RPEM.

In a preferred embodiment of the invention, the computations according to the SAEKF are executed as summarised by the following steps. Numerous other implementations of the SAEKF are possible, with different numerical accuracy, effort and stability.

1. State estimate propagation, solved e.g. by numeric integration of the complete state equations for the time t in the range $t_k \leq t \leq t_{k+1}^-$:

$$\dot{\hat{x}} = f(\hat{x}, u, \hat{p}(t_k))$$

2. Covariance propagation, solved either by numeric integration or by explicit solution of the differential equations:

$$\dot{P}_{xx} = \frac{\partial f}{\partial x^{\mathrm{T}}} P_{xx} + P_{xx} \frac{\partial f^{\mathrm{T}}}{\partial x} + \frac{\partial f}{\partial p^{\mathrm{T}}} P_{xp}^{\mathrm{T}} + P_{xp} \frac{\partial f^{\mathrm{T}}}{\partial p} + Q$$

$$\dot{P}_{xp} = \frac{\partial f}{\partial x^{\mathrm{T}}} P_{xp} + \frac{\partial f}{\partial p^{\mathrm{T}}} P_{pp}$$

$$\dot{P}_{pp} = Q_{pp}$$

3. Predicted output computation:

$$\hat{y}(t_{k+1}) = h\left(\hat{x}(t_{k+1}^-), u(t_{k+1}), \hat{p}(t_k)\right)$$

4. Prediction error computation:

$$e(t_{k+1}) = y(t_{k+1}) - \hat{y}(t_{k+1})$$

5. Approximate prediction of error covariance matrix computation:

$$A(t_{k+1}) = R + \frac{\partial h}{\partial x^{T}} P_{xx}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial x} + \frac{\partial h}{\partial x^{T}} P_{xp}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial p}$$

$$+ \frac{\partial h}{\partial p^{T}} P_{xp}^{T}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial x} + \frac{\partial h}{\partial p^{T}} P_{pp}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial p}$$

6. Computation of Filter gain matrices, where K is the Kalman Filter Gain for the state x and L is the Kalman Filter Gain for the augmented state p:

$$K(t_{k+1}) = (P_{xx}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial x} + P_{xp}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial p}) A^{-1}(t_{k+1})$$

$$L(t_{k+1}) = (P_{xp}^{T}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial x} + P_{pp}(t_{k+1}^{-}) \frac{\partial h^{T}}{\partial p}) A^{-1}(t_{k+1})$$

7. State update computation:

$$\Delta\hat{x}(t_{k+1}) = K(t_{k+1}) e(t_{k+1})$$

$$\hat{x}(t_{k+1}^{+}) = \hat{x}(t_{k+1}^{-}) + \Delta\hat{x}(t_{k+1})$$

8. Parameter Update computation:

$$\Delta\hat{p}(t_{k+1}) = L(t_{k+1}) e(t_{k+1})$$

$$\hat{p}(t_{k+1}) = \hat{p}(t_{k}) + \Delta\hat{p}(t_{k+1})$$

9. Covariance update computation:

$$P_{xx}(t_{k+1}^{+}) = P_{xx}(t_{k+1}^{-}) - K(t_{k+1})A(t_{k+1})K^{T}(t_{k+1})$$

$$P_{xp}(t_{k+1}^{+}) = P_{xp}(t_{k+1}^{-}) - K(t_{k+1})A(t_{k+1})L^{T}(t_{k+1})$$

$$P_{pp}(t_{k+1}^{+}) = P_{pp}(t_{k+1}^{-}) - L(t_{k+1})A(t_{k+1})L^{T}(t_{k+1})$$

The above steps are performed iteratively, for increasing time t and with measured variables u. What makes the SAEKF work, is the connection of the variables p and the system states x through the covariances of the noise processes.

Turbomachine application:

A compressor map typically describes a dependency of the mass flow $\dot{m}$ of a working fluid through a compressor or turbine, and/or an efficiency e of the compressor or turbine process. Usually, a pressure ratio, a rotational speed and optionally a guide vane angle are measured or estimated, and from these the mass flow and efficiency are determined. This is done either manually from a graphic representation of the map, or in a control system according to look-up tables and computations, or from polynomial approximations to the respective functions. The dependency of the efficiency and mass flow on the known variables is therefore static and nonlinear. Due to degradation of the machine, e.g. by blade erosion, the dependency changes over time, and a continuously updated estimate of the dependency is desired.

For this estimation, the traditional approach is to model the dependency of mass flow on the system state as

$$\dot{m}' = \dot{m}'_{ref} C_1(w) C_2(v)$$
$$C_1(w) = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + \cdots$$
$$C_2(v) = b_0 + b_1 v + b_2 v^2 + b_3 v^3 + \cdots$$

where $\dot{m}'$ is a normalised mass flow, w represents rotational speed and v represents a guide vane angle. From $\dot{m}'$ and the pressure ratio, the estimate of the actual mass flow $\dot{m}$ is computed. Alternatively, the dependency on the pressure ratio dP can be expressed as a third polynomial as illustrated below.

$$\dot{m} = \dot{m}_{ref} C_1(w) C_2(v) C_3(dP)$$
$$C_1(w) = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + \cdots$$
$$C_2(v) = b_0 + b_1 v + b_2 v^2 + b_3 v^3 + \cdots$$
$$C_3(dP) = c_0 + c_1 dP + c_2 dP^2 + c_3 dP^3 + \cdots$$

The SAEKF is applied to estimate the polynomial coefficients $a_0 \ldots a_n$ and $b_0 \ldots b_n$. From a current estimate of said coefficients and from current values of w, v and pressure ratio, the mass flow $\dot{m}$ is determined. An analogous procedure is applied for the efficiency e.

According to the invention, it is not necessary to have an explicit representation of the dependency of $\dot{m}$ and e on the system state. It is sufficient to know that there exists such a dependency. The dependency can be either linear or nonlinear, static or with memory. An example of a nonlinear

dependency with memory is a backlash model of a gear transmission as represented by Figure 6.

According to the invention, the variables $\dot{m}$ and e are considered as *augmented states* of the corresponding compressor or turbine model. The SAEKF approach, or a suitable modification, is then used to estimate the augmented states. The SAEKF then tracks the mass flow $\dot{m}$ and efficiency e, although they change both relatively fast with the operating state of the machine, and relatively slow with a degradation of the machine.

In a preferred embodiment of the invention, the variables estimated in this way are influenced by conditions that change slowly with respect to the main process dynamics, e.g. turbine degradation due to erosion and fouling. The efficiency estimate under the same operating conditions shall deteriorate over time, and its value can therefore be used as a basis for preventive maintenance, equipment health monitoring, fault diagnosis, etc.

The operating state or condition undergoes changes with time constants in the range of less than a second to several minutes, whereas the degradation has a noticeable effect over weeks and months. The traditional approach would only compensate for the degradation, whereas the state changes would be taken into account through the computation of $\dot{m}$ and e from the state through the polynomially approximated functions.

In another preferred embodiment of the invention, an explicit representation of the dependency of $\dot{m}$ and e is determined from a parametric fitting of the estimates provided by the SAEKF and the corresponding measured or estimated values of the arguments such as speed, pressure ratio, etc. The explicit model then can be used in offline "what-if" simulations.

The representation according to equation (1), as used in the SAEKF is obtained in the following way: Figure 3 shows a block diagram representing the computation of process variables in a flow process. Arrows entering a block denote input variables to the block, from which the block computes output variables denoted by outgoing arrows. N1 is a flow element which computes a flow F1 through e.g. a turbine or compressor and an exit temperature T1i from inlet pressure P0, outlet pressure P1, inlet temperature T0, rotational speed w and guide vane angle v. The computation corresponds to nonlinear and static, i.e. memoryless functions. Analogous relations hold for flow element N2. A flow element may also represent a valve, with an input corresponding to valve stroke instead of w and v, or simply part of a pipe.

V1 is a storage element, in which a pressure P1 and temperature T1o in the element are computed from inlet temperature T1i and from an inflow F1 and outflow F2. These computations are based on mass and energy balances in the storage element, and represent the dynamic part of the process. In all relations, enthalpies may be used instead of or in addition to temperatures. Flows are mass flows $\dot{m}$ or volume flows.

Flow and storage elements are daisy-chained as in Figure 3 to represent a complete compressor or turbo-group together with preceding and subsequent piping. In order to obtain the state-space representation of equation (1), the states of the storage elements are collected in the state vector **x**, and the flows and optionally the efficiencies are collected in the augmented state vector **p**. In the vector equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p})$, the vector function **f** represents the modelled dynamics of all the storage elements V1,V2,... Such dynamic models for turbines and thermohydraulic processes are commonly known. According to the invention, the static relations embodied by the flow elements N1,N2,... and leading to the vector of variables **p** are not modelled explicitly. Instead, the variables in **p**, e.g. the flows, are determined by the SAEKF. The derivatives of the variables in **p** are set to zero, as expressed by equation (1), and their values are updated by the SAEKF through the effect of the covariance matrices P_{xx}, P_{xp}, P_{pp}.

Another example of a polynomial approximation is found in Knud Rasmussen, "Calculation methods for the physical properties of air used in the calibration of microphones", Department of acoustic technology, Technical university of Denmark report PL-11b, 1997 (http://www.dat.dtu.dk/docs/PL11b-RAP.PDF) where the specific heat ratio of humid air is approximated as follows:

$$\begin{aligned}
\kappa &= a_0 + a_1 T + a_2 T^2 + (a_3 + a_4 T + a_5 T^2) x_w + (a_6 + a_7 T + a_8 T^2) P + a_9 x_w^2 + a_{10} P^2 \\
x_w &= \frac{H}{100} \frac{P_{sv}(T)}{P} f(P,T) \\
P_{sv}(T) &= exp(b_0 T^2 + b_1 T + b_2 + b_3 T^{-1}) \\
f(P,T) &= c_0 + c_1 P + c_2 T^2
\end{aligned}$$

where a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, b_0, b_1, b_2, b_3, c_0, c_1, c_2 are polynomial coefficients to be estimated. In the formulas above, κ is the specific heat ratio, T is temperature, P is pressure, x_w is a mole fraction of water vapour in air, Psv is the saturation water pressure and H is the relative humidity in percentage. According to the invention, the polynomial for the specific heat capacity κ is replaced with one single augmented state and included in a larger model, for example in a model of a compressor, a turbine or a heat exchanger.

Heat exchanger application:

Figure 4 illustrates a model for a heat transfer process in a heat exchanger pipe or pipe section: Hot exhaust gas with an inlet temperature T_{gi} and mass flow rate f_g transfers energy to a metal pipe wall with temperature T_m. The energy is further transferred from the pipe to steam with an inlet temperature T_{si} and flow rate f_s. The gas temperature is changed to T_g and the steam temperature to T_s. The following equations may be used to model the process, where heat transfer coefficients from gas to metal and metal to steam are denoted by α_g and α_s, respectively:

$$\dot{T}_g = \alpha_g (T_m - T_g) + f_g (T_{gi} - T_g)$$
$$\dot{T}_m = \alpha_g (T_g - T_m) + \alpha_s (T_s - T_m)$$
$$\dot{T}_s = \alpha_s (T_m - T_s) + f_s (T_{si} - T_s)$$

The inlet temperatures T_{si}, T_{gi} are assumed to be known, the temperatures T_g, T_m, T_s are model states. Alternatively, enthalpy is used as a steam state and

steam temperature is computed from steam tables. Some or all of the states are measured, with measurement noise vector v.

The known approach would be to represent α_g and α_s by polynomials in terms of the flow rates f_g and f_s. , i.e.

$$\alpha_g = \alpha_0 + \alpha_1 f_g + \alpha_2 f_g^2 + \ldots$$
$$\alpha_s = \beta_0 + \beta_1 f_s + \beta_2 f_s^2 + \ldots$$

and to estimate the polynomial coefficients α_0 , α_1 , ... and β_0 , β_1 with a SAEKF. According to the invention, the heat transfer coefficients α_g and α_s themselves are incorporated as augmented states in the vector of variables p and estimated by the SAEKF.

Gear backlash application:

The differential equations of motion for an exemplary two-mass motor-arm system in a robot, schematically shown in Figure 5, are as follows.

$$\begin{aligned}
\dot{x_1} &= x_3 \\
\dot{x_2} &= x_4 \\
\dot{x_3} &= \frac{1}{m_1}(u - D(x_3 - x_4) - \tau_K - \tau_f - K_r) \\
\dot{x_4} &= \frac{1}{m_2}(D(x_3 - x_4) + \tau_K) \\
\tau_f &= C_v x_3 + C_c \frac{\pi}{2} \operatorname{atan}(\alpha x_3)
\end{aligned}$$

where x_1, x_2 are motor and arm positions, and m_1, m_2 are motor and arm inertias, respectively. τ_K is a nonlinear backlash and spring function to be identified. K_r represents a proportional or P-controller on the motor position, u

is an additional torque input used for identification, D is a known damper coefficient and τ_f is a motor friction torque. The model is written on short form as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}) + v$$

where p=[τK]T is the augmented state and the unknown backlash and spring function to be estimated. Figure 6 shows a backlash torque τk with memory effect as a function of a position difference x1-x2 between the motor M and the arm A. τ_f is the viscous and Coloumb friction torque model. To get the Kalman filter to work properly, a continuous approximation of the Coloumb friction is needed in order to compute the gradients df/dx at speeds close to zero. The friction torque as represented by the above equation for τ_f is illustrated in Figure 8.

The measurements are either motor velocity only or both motor and arm velocity. The two candidates for the measurement vector y equal

$$\begin{aligned} y_1 &= x_3 + w \\ y_2 &= [x_3 + w_1, x_4 + w_2]^T \end{aligned}$$

where w, w_1 and w_2 are noise processes. The complete model is illustrated in Figure 5.

One advantage of the State-Augmented filter approach according to the invention is the fact that it is not necessary to assume a particular model structure for the backlash and spring function to be identified. Instead, τ_k is simply made an augmented state.

The augmented states are introduced into the system equations in the following way.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}) \\ 0 \end{bmatrix} + v$$

Traditionally, polynomials have been used to approximate nonlinear functions and the constant coefficients of the polynomials are then selected as the augmented states. An example 3rd order polynomial $y=\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$ is shown by the curved trajectory in Figure 7 for the approximation of a linear spring with backlash according to the piecewise linear trajectory.

The polynomial approach for approximating the nonlinear function has several drawbacks as listed below.

- The polynomial is a continuous approximation and it is difficult to estimate the size of the backlash from the coefficients α_0, α_1, α_2, α_3.
- The polynomial approach introduces unnecessarily additional augmented states and increases the complexity of the problem.

One significant advantage of the augmented Kalman filter is the fact that the parameters to be estimated do not have to be constants. The polynomial coefficients are replaced by a single augmented state for the spring and backlash force τ_K. The augmented filter then estimates the force τ_K as a function of time and no particular shape of the underlying nonlinearity has to be assumed.

The complete augmented model can be written as follows.

$$
\begin{aligned}
\dot{x_1} &= x_3 + v_1 \\
\dot{x_2} &= x_4 + v_2 \\
\dot{x_3} &= \frac{1}{m_1}\left(u - D(x_3 - x_4) - p - \tau_f - K_r\right) + v_3 \\
\dot{x_4} &= \frac{1}{m_2}\left(D(x_3 - x_4) + p\right) + v_4 \\
\dot{p} &= v_5
\end{aligned}
$$

where variables v_i are noise processes acting on the state and parameter variables. The covariance matrix for the augmented noise process v is given as

$$E(v(t)v^T(t')) = \begin{bmatrix} Q_{xx} & 0 \\ 0 & Q_{pp} \end{bmatrix} \delta(t - t')$$

where Q_{xx} is the covariance of the noise on the state derivatives, while Q_{pp} is the covariance of the noise on the augmented state derivative, ie. the derivative of the backlash and spring force. Q_{pp} can be seen as the main tuning parameter of the augmented filter. If Q_{pp} is set to zero, the parameter p will remain a constant equal to its initial value. The larger the value of Q_{pp}, the faster the augmented state will be updated.

List of designations

A	Arm
D	damper coefficient
τ_f	motor friction torque
K_r	force from P-controller on the motor position
M	Motor
u	additional torque input
τ_K	nonlinear backlash and spring function